A METHOD OF CALCULATING THE RADIATION FLUX AND ITS DIVERGENCE IN REGIONS WITH STEPPED TEMPERATURE AND CHEMICAL COMPOSITION DISTRIBUTION

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An approximate method is proposed which makes it possible to compute the radiation flux and its divergence when the absorption cross section depends in a complex manner on the wavelength. Radiation scattering is ignored. The method is described for the case when the region occupied by the radiating and absorbing gas can be divided into finite number of subregions in which the temperature and the chemical composition are constant. Examples are given of the numerical calculations of the radiation flux.

1. Radiation Heat Flux from a Two-Dimensional Layer. In problems in radiation gas dynamics we have to solve simultaneously the fundamental system of equations of motion and the radiation transport equation, the integration of which gives an expression for the radiation heat flux and its divergence. For a two-dimensional radiating layer of gas, assuming the temperature and chemical composition dependent only on the transverse coordinate y, we have the following expression [1, 2] for the radiation heat flux at the surface of the body (y = 0):

$$q_w = 2\pi \int_{(\Delta\lambda)} d\lambda \int_0^{t_\lambda(\Delta)} B_\lambda(y) E_2[t_\lambda(y)]\, dt_\lambda(y), \qquad t_\lambda(y) = \int_0^y k_\lambda\, dy' \tag{1.1}$$

Here it is assumed that radiation does not fall on the layer from the outside, Δ is the thickness of the layer, $t_\lambda(y)$ is the optical distance of the point with coordinate y from the surface of the layer taken as the origin for the coordinate, $E_n(t)$ is the exponential integral function of the n-th order (n = 1, 2, 3), B_λ is Planck's equilibrium radiation function, $(\Delta\lambda)$ is the wavelength band within which the radiation falls, k_λ is the reduced volume coefficient of absorption taking into account forced emission depending on the temperature, pressure, and component concentration.

Thus, the optical distance t_λ can be computed only after we know the gas dynamic field. In actual problems k_λ is a rapidly oscillating function of the wavelength λ, and so the computation of the integral with respect to λ in (1.1) is difficult.

The method we propose is based on the following two transformations:

1. In the radiating layer we replace the continuous temperature, pressure, and component molar concentration distributions by stepped concentrations by dividing the radiating layer into $\varkappa$ elementary layers of thicknesses $(\Delta y)_j$ ($j = 1, \ldots, \varkappa$), inside which the temperature T, the pressure p, and the concentration x_i are constant (Fig. 1). This transformation is partly constrained and partly natural as indicated by the following considerations:

a) at the present time there are no detailed tables for the functions k_λ (T, p, x_i) of many variables but there is a set of functions k_λ(T, p) of λ for some equilibrium mixtures when the temperature and pressure steps are sufficiently coarse;

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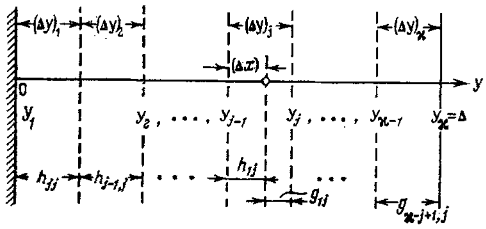

Fig. 1

b) moreover, the solutions of many problems in gas dynamics may indeed have profiles of T, p, and x_i which are nearly stepped.

2. We approximate the integrals I_n

$$I_n(t) = n\int_0^t E_n(t)\,dt = 1 - nE_{n+1}(t) \qquad (n = 1, 2) \tag{1.2}$$

$$I_0 = \int_0^t e^{-t}\,dt = 1 - e^{-t}$$

as follows:

$$\begin{aligned} I_n(t) &= S_{n\nu} = a_1 t + a_2 t^2 + \ldots + a_\nu t^\nu \quad \text{for} \quad t \leqslant t_{(\nu)} \\ I_n(t) &= 1 \quad \text{for} \quad t > t_{(\nu)} \end{aligned} \tag{1.3}$$

To illustrate, we give expressions for the approximating polynomials $S_{0\nu}$, $S_{1\nu}$, $S_{2\nu}$ corresponding to I_0, I_1, I_2 having $\nu = 1, 2, 3$ terms.

$$\begin{aligned}
S_{01} &= 0.69t && (t_{(\nu)} = 1.44,\ \varepsilon = 0.31) \\
S_{02} &= 0.8696t - 0.189t^2 && (t_{(\nu)} = 2.3,\ \varepsilon = 0.14) \\
S_{03} &= 0.93t - 0.29t^2 + 0.03t^3 && (t_{(\nu)} = 4,\ \varepsilon = 0.097) \\
S_{11} &= 3.3t && (t_{(\nu)} = 0.29,\ \varepsilon = 0.92) \\
S_{12} &= 3.3t - 2.72t^2 && (t_{(\nu)} = 0.6,\ \varepsilon = 0.45) \\
S_{13} &= 3.3t - 4.65t^2 + 2.25t^3 && (t_{(\nu)} = 1.1,\ \varepsilon = 0.18) \\
S_{21} &= 1.22t && (t_{(\nu)} = 0.81,\ \varepsilon = 0.39) \\
S_{22} &= 1.56t - 0.6084t^2 && (t_{(\nu)} = 1.3,\ \varepsilon = 0.22) \\
S_{23} &= 1.84t - 1.376t^2 + 0.368t^3 && (t_{(\nu)} = 1.8,\ \varepsilon = 0.113)
\end{aligned}$$

Here we indicate the values of $t_{(\nu)}$ and of the relative error ε in the approximation in parentheses.

The coefficients a_s of the function I_2 are chosen so that the relative error in the approximation should be minimal for $t \leq t_{(\nu)}$. We note that even when the degree of the polynomial is small ($\nu = 2$ or 3) the accuracy of the approximation is quite satisfactory, while as ν increases the error can be made arbitrarily small. Figure 2 shows the graph of $I_2(t)$ (continuous line) and of its approximations for $\nu = 1, 2, 3$ (dotted lines). For simplicity in describing the method we shall consider only the case when absorption is determined by bound-bound and bound-free transitions. Then $k_{\lambda j}$, $\sigma_{\lambda j}$, and the number of particles in unit volume N_j in layer j are linked by the equation

$$k_{\lambda j} = N_j \sigma_{\lambda j}$$

Free-free transitions contributing to k_λ proportionally to the product of the ion and the electron concentrations can easily be taken into account by a small modification of the method.

Let $(\Delta t_\lambda)_j$ and $t_{\lambda i}$ denote respectively the optical thickness of an elementary layer j and the sum of i elementary layers from the first through the i-th, and let n_j denote the number of particles in the j-th layer falling on unit area of the surface $y = 0$

$$t_{\lambda i} = \sum_{j=1}^{i} (\Delta t_\lambda)_j, \quad (\Delta t_\lambda)_j = n_j \sigma_{\lambda j}, \quad n_j = (\Delta y)_j N_j \tag{1.4}$$

Taking the first transformation into account, we can write (1.1) as

$$q_w = \sum_{j=1}^{\varkappa} z_j \pi B(T_j) = \sum_{j=1}^{\varkappa} z_j \sigma^* T_j^4 \tag{1.5}$$

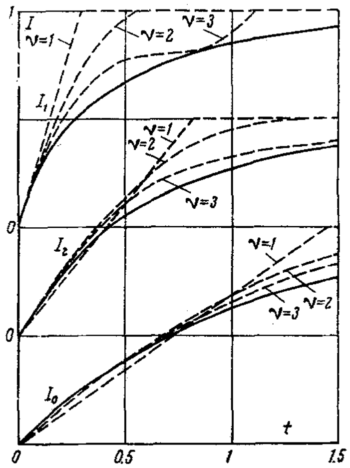

Fig. 2

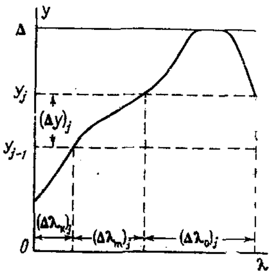

Fig. 3

$$z_j = \int\limits_{(\Delta\lambda)} \left[B_{\lambda j}^{\circ} 2 \int\limits_{(\Delta y)_j} E_2(t_\lambda)\, dt_\lambda \right] d\lambda, \quad B_{\lambda j}^{\circ} = \frac{B_{\lambda j}}{B(T_j)}, \quad B(T_j) = \frac{\sigma^*}{\pi} T_j^4 \tag{1.6}$$

Here $\pi B(T_j)$ $(j = 1, \ldots, \varkappa)$ is the radiation heat flux from an absolutely black radiating layer at temperature T_j and the coefficient $z_j \le 1$ $(j = 1, \ldots, \varkappa)$ takes into account the difference between the layer and an absolutely black one and the attenuation of this radiation in the layer $1, 2, \ldots, j-1$; σ^* is the Stefan–Boltzmann constant.

For the first layer, z_1 indicates the degree of blackness.

Consider the graph (Fig. 3) of the function $y_{(\nu)}(\lambda)$, defined in implicit form in the λy-plane by the equation

$$\int\limits_0^{y_{(\nu)}} \sigma_\lambda(y) N(y)\, dy = t_{(\nu)}$$

where $t_{(\nu)}$ is fixed. The curve $y = y_{(\nu)}(\lambda)$ is the geometrical locus of points at the same optical distance $t_{(\nu)}$ from the wall $(y = 0)$ and it divides the whole region $[(\Delta\lambda), 0 \le y \le \Delta]$ into two: in one part $t_\lambda \le t_{(\nu)}$, while in the other $t_\lambda > t_{(\nu)}$. To take into account the radiation which reaches the wall from the layer numbered j, illustrated in Fig. 3, we divide the whole range of wavelengths $(\Delta\lambda)$ for each layer j into three: $(\Delta\lambda_k)_j$, $(\Delta\lambda_m)_j$, $(\Delta\lambda_0)_j$ such that

$$\begin{aligned} &\lambda \in (\Delta\lambda_k)_j \quad \text{if} \quad t_{(\nu)} \leqslant t_{\lambda, j-1} \\ &\lambda \in (\Delta\lambda_m)_j \quad \text{if} \quad t_{\lambda, j-1} < t_{(\nu)} < t_{\lambda j} \\ &\lambda \in (\Delta\lambda_0)_j \quad \text{if} \quad t_{\lambda j} \leqslant t_{(\nu)} \end{aligned} \tag{1.7}$$

The range $(\Delta\lambda_k)_j$ of wavelengths does not make any contribution to the radiation flux at the wall from layer j since, in view of the approximation of the integrals (1.3), the radiation in these wavelengths is completely absorbed by the layers $(i = 1, \ldots, j-1)$ nearer to the wall. We note that $(\Delta\lambda_k)_1 = 0$ since the whole graph of the function $y = y_{(\nu)}(\lambda)$ lies above the λ axis. From the range $(\Delta\lambda_m)_j$ of wavelengths only the radiation from that part of the layer $(\Delta y)_j$ for which $y < y_{(\nu)}$ reaches the wall. Radiation from all points of the layer $(\Delta y)_j$ reaches the wall from the range $(\Delta\lambda_0)_j$ of wavelengths. We note that if we take into account the actual absorption spectrum, in general we obtain a function $\lambda = y_{(\nu)}^{-1}(y)$ which is not single-valued in the layer $(\Delta y)_j$.

Hence, some or all of the ranges $(\Delta\lambda_k)_j$, $(\Delta\lambda_m)_j$, $(\Delta\lambda_0)_j$ of wavelengths consist of a large number of intervals.

Using the approximation (1.3) and the subdivision (1.7) of $(\Delta\lambda)$, we obtain

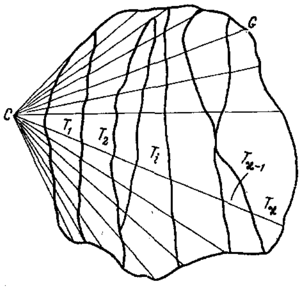

Fig. 4

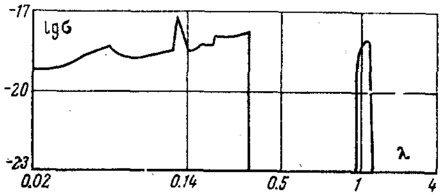

Fig. 5

$$
\begin{gathered}
\Phi_l = 2 \int\limits_{(\Delta y)_l} E_2 dt_\lambda \qquad (l = 1, \ldots, \varkappa;\ j = 2, \ldots, \varkappa) \\
\Phi_1 = \sum_{s=1}^{\nu} a_s (\Delta t_\lambda)_1{}^s \ \text{ for } \ \lambda \in (\Delta\lambda_0)_1 \\
\Phi_1 = 1 \qquad \text{for } \ \lambda \in (\Delta\lambda_m)_1 \\
\Phi_j = \sum_{s=1}^{\nu} a_s \sum_{k=0}^{s-1} \binom{s}{k} t^k_{\lambda, j-1} (\Delta t_\lambda)_j^{s-k} \quad \text{for } \ \lambda \in (\Delta\lambda_0)_j \\
\Phi_j = 1 - \sum_{s=1}^{\nu} a_s t^s_{\lambda, j-1} \qquad \text{for } \ \lambda \in (\Delta\lambda_m)_j \\
\Phi_j = 0 \qquad \text{for } \quad \lambda \in (\Delta\lambda_k)_j \\
\left(\binom{s}{k} - \text{binomial coefficient} \right)
\end{gathered}
\tag{1.8}
$$

Finally, noting (1.8) and (1.4) we obtain the following equations for the coefficients z_j ($j = 1, \ldots, \varkappa$) of (1.6):

$$
z_1 = \sum_{s=1}^{\nu} a_s n_1{}^s M^{(1s0)} + M^{(100)}
$$

$$
z_j = \sum_{s=1}^{\nu} a_s \sum_{k=0}^{s-1} \binom{s}{k} n_j^{s-k} \sum_{i_1=1}^{j-1} \cdots \sum_{i_k=1}^{j-1} n_{i_1} \ldots n_{i_k} M^{(jsk)}_{i_1, \ldots, i_k} + M^{(j00)} - \sum_{s=1}^{S} a_s \sum_{i_1=1}^{j-1} \cdots \sum_{i_s=1}^{j-1} n_{i_1} \ldots n_{i_s} M^{(jss)}_{i_1, \ldots, i_s} \quad (j \geqslant 2) \tag{1.9}
$$

$$
\begin{gathered}
M^{(jsk)}_{i_1, \ldots, i_k} = \int\limits_{(\Delta\lambda_0)_j} \overset{\circ}{B}_{\lambda j} \sigma_{\lambda i_1} \ldots \sigma_{\lambda i_k} \sigma_{\lambda j}^{s-k} d\lambda \\
M^{(j00)} = \int\limits_{(\Delta\lambda_m)_j} \overset{\circ}{B}_{\lambda j} d\lambda, \qquad M^{(jss)}_{i_1, \ldots, i_s} = \int\limits_{(\Delta\lambda_m)_j} \overset{\circ}{B}_{\lambda j} \sigma_{\lambda i_1} \ldots \overline{\sigma_{\lambda i_s}} d\lambda \\
(j = 1, \ldots, \varkappa;\ i_l = 1, \ldots, j-1;\ s = 1, \ldots, \nu;\ a = 0, \ldots, s-1)
\end{gathered}
\tag{1.10}
$$

When $k = 0$ the product $\sigma_{\lambda i_1} \ldots \sigma_{\lambda i_k}$ in the integrals for M vanishes. In a problem with two radiating layers ($\varkappa = 2$) the subscripts $i_1, \ldots, i_k$ can only take the single value unity. In such problems the subscripts may be dropped from M.

In general the integrals $M^{(jsk)}$ depend on the temperature, the number of particles, and the composition of the mixture in the elementary layers (Δy). They are the analogs of the Planck mean absorption coefficient k_p

$$
k_P = \int\limits_{(\Delta\lambda)} k_\lambda B_\lambda{}^\circ d\lambda = N \int\limits_{(\Delta\lambda)} B_\lambda{}^\circ \sigma_\lambda d\lambda
$$

and contain information about the optical properties of the layers (Δy).

When $s = k = 1$ and $t_\lambda \leq t_{(\nu)}$, for all $\lambda \in (\Delta\lambda)$ the integral $M^{(jsk)}$ has the value

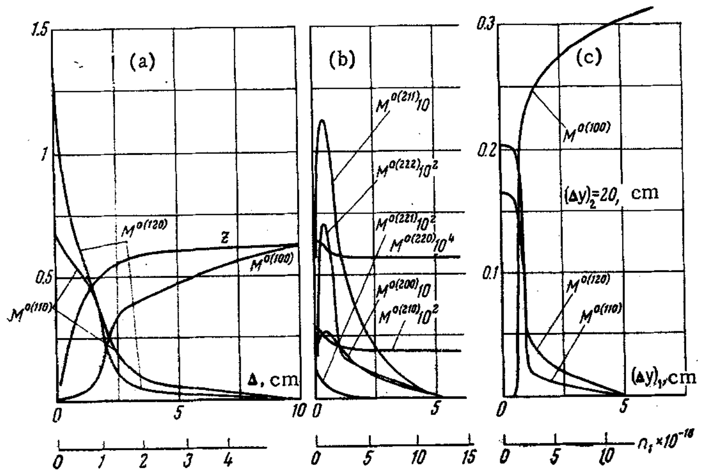

Fig. 6

$$M^{(j11)} = \int_{(\Delta\lambda)} B_\lambda{}^\circ \sigma_{\lambda j} d\lambda$$

which is proportional to k_p. By introducing the Planck mean absorption coefficient we avoid integration with respect to λ when solving problems in the limiting case of an optically thin radiating region. By introducing the integrals $M^{(jsk)}$ we can do this for a radiating layer of any optical thickness.

If the integrals $M^{(jsk)}(n_1, \ldots, n_j)$ are known for $t_{(\nu)} = \tau$, then for any other value $t_{(\nu)} = \tau_*$ the corresponding integrals, which we denote by $M_*^{(jsk)}(n_1, \ldots, n_j)$, are obtained from the integrals $M^{(jsk)}$ by changing the scale of the variables $n_1, \ldots, n_j$

$$M_*^{(jsk)}(n_1, \ldots, n_j) = M^{(jsk)}(n_1\tau_*/\tau, \ldots, n_j\tau_*/\tau) \tag{1.11}$$

2. Divergence of the Radiation Heat Flux in a Two-Dimensional Layer. The expression for the divergence of the radiation flux at an internal point of a two-dimensional layer has the form [2]

$$\operatorname{div} q(y) = \int_{(\Delta\lambda)} d\lambda k_\lambda(y) \left\{ \int_0^{t_\lambda(\Delta)} 2\pi B_\lambda(v) E_1[\,|t_\lambda(v) - t_\lambda(y)|\,]\, dt_\lambda(v) - 4\pi B_\lambda(y) \right\} \tag{2.1}$$

Here we assume that no radiation falls on the layer from outside.

The difference from the case in which the radiation flux q_w at the wall is computed is that div q(y) has to be computed at points internal to the elementary layers $(\Delta y)_j$.

We divide the radiating layer into elementary layers $(\Delta y)_j$ $(j = 1, \ldots, \varkappa)$ in accordance with the first transformation of § 1.

We compute the function div q(y) at the point $y \in (\Delta y)_j$ at a distance (Δx) from the point y_{j-1}. We introduce new notation for the elementary layers, using Greek letters to enumerate them:

$$g_{\alpha j} = \begin{cases} (\Delta y)_j - (\Delta x) & \text{for } \alpha = 1 \\ (\Delta y)_{\alpha+j-1} & \text{for } \alpha = 2, \ldots, \varkappa - j + 1 \end{cases}$$

$$h_{\beta j} = \begin{cases} (\Delta x) & \text{for } \beta = 1 \\ (\Delta y)_{j-\beta+1} & \text{for } \beta = 2, \ldots, j \end{cases}$$

The new enumeration is shown in the lower part of Fig. 1. For $y \in (\Delta y)_j$, Eq. (2.1) takes the form

$$\operatorname{div} q(y) = 2\pi \sum_{\alpha=1}^{\varkappa-j+1} w_{\alpha j} B(T_\alpha) + 2\pi \sum_{\beta=1}^{j} \mathring{w}_{\beta j} B(T_\beta) - 4u_j B(T_j)$$

$$w_{\alpha j} = \int\limits_{(\Delta\lambda)} d\lambda \mathring{B}_{\lambda\alpha} N_j \sigma_{\lambda j} \int\limits_{g_{\alpha j}} E_1 [t_\lambda (g)]\, dt_\lambda (g)$$

$$\mathring{w}_{\beta j} = \int\limits_{(\Delta\lambda)} d\lambda \mathring{B}_{\lambda\beta} N_j \sigma_{\lambda j} \int\limits_{h_{\beta j}} E_1 [t_\lambda (h)]\, dt_\lambda (h), \quad u_j = N_j \int\limits_{(\Delta\lambda)} \mathring{B}_{\lambda j} \sigma_{\lambda j} d\lambda$$

Here we have introduced new coordinates g and h to denote the distance of the current point y' from the point y at which div q(y) is computed

$$g = y' - y \ \text{for} \ \ y' > y; \qquad h = y - y' \ \text{for} \ \ y > y'$$

We denote the coefficients a_s in the representation (1.3) from I_1 by a_s'. The coefficients a_s' are chosen so that I_1 is approximated with minimal relative error ε by a polynomial in the interval $(t_{(m)})$, $(t_{(\nu)})$. The values of a_s', ε, $t_{(\nu)}$ are given in § 1 for the start of the approximation interval $t_{(m)} = 0.05$. In Fig. 2 the continuous line is the exact function $I_1(t)$ and the dotted lines are approximations to it.

Finally we have

$$w_{1j} = \sum_{s=1}^{\nu} a_s' \overset{s}{n}_{(\alpha=1)} N_j M^{(1s0j)} + N_j M^{(100j)}$$

$$w_{\alpha j} = \sum_{s=1}^{\nu} a_s' \sum_{k=0}^{s-1} \binom{s}{k} N_j n_\alpha^{s-k} \sum_{\alpha_1=1}^{\varkappa-j} \ldots \sum_{\alpha_k=1}^{\varkappa-j} n_{\alpha_1} \ldots n_{\alpha_k} M^{(\alpha skj)}_{\alpha_1, \ldots, \alpha_k} + N_j M^{(\alpha 00j)}$$

$$- \sum_{s=1}^{\nu} a_s' \sum_{\alpha_1=1}^{\varkappa-j} \ldots \sum_{\alpha_s=1}^{\varkappa-j} N_j n_{\alpha_1} \ldots n_{\alpha_s} M^{(\alpha ssj)}_{\alpha_1, \ldots, \alpha_s}$$

$$u_j = N_j M^{(j00j)}_{(\Delta\lambda)}, \quad \mathring{w}_{1j} = \sum_{s=1}^{\nu} a_s' \overset{s}{n}_{(\beta=1)} N_j M^{(1s0j)} + N_j M^{(100j)}$$

$$\mathring{w}_{\beta j} = \sum_{s=1}^{\nu} a_s' \sum_{k=0}^{s-1} \binom{s}{k} N_j n_\beta^{s-k} \sum_{\beta_1=1}^{j-1} \ldots \sum_{\beta_k=1}^{j-1} n_{\beta_1} \ldots n_{\beta_k} M^{(\beta skj)}_{\beta_1, \ldots, \beta_k} + N_j M^{(\beta 00j)}$$

$$- \sum_{s=1}^{\nu} a_s' \sum_{\beta_1=1}^{j-1} \ldots \sum_{\beta_s=1}^{j-1} N_j n_{\beta_1} \ldots n_{\beta_s} M^{(\beta ssj)}_{\beta_1, \ldots, \beta_s}$$

where

$$M^{(iskj)}_{i_1, \ldots, i_k} = \int\limits_{(\Delta\lambda_0)_{ij}} \mathring{B}_{\lambda i} \sigma_{\lambda i_1} \ldots \sigma_{\lambda i_k} \sigma_{\lambda j}^{s-k} \sigma_{\lambda j} d\lambda, \quad M^{(j00j)}_{(\Delta\lambda)} = \int\limits_{(\Delta\lambda)} \mathring{B}_{\lambda j} \sigma_{\lambda j} d\lambda$$

$$M^{(i00j)} = \int\limits_{(\Delta\lambda_m)_{ij}} \mathring{B}_{\lambda i} \sigma_{\lambda j} d\lambda, \quad M^{(issj)}_{j_1, \ldots, j_s} = \int\limits_{(\Delta\lambda_m)_{ij}} \mathring{B}_{\lambda i} \sigma_{\lambda i_1} \ldots \sigma_{\lambda i_s} \sigma_{\lambda j} d\lambda$$

$$(i_l = \alpha_1, \ldots, \alpha_{\varkappa-j+1}, \beta_1, \ldots, \beta_j)$$

The subscript i is computed from α or β by means of the equations

$$i = j + \alpha - 1, \qquad i = j - \beta + 1$$

The regions of integration $(\Delta\lambda_0)_{ij}$ and $(\Delta\lambda_m)_{ij}$ are defined by equations similar to (1.7), taking into account the optical distance between y_i and y.

3. Three-Dimensional Radiation Transport. The ideas behind the transformation of Sec. 1 are independent of the number of dimensions of the space filled with the radiating gas and so the method is appropriate for computing the radiation flux and its divergence in two- and three-dimensional radiation transport problems. For simplicity we consider the case of the computation of the radiation flux across an area at a point C from a region G containing a one-component gas.

For simplicity in exposition we shall assume that the temperature distribution along each ray passing through the point C and intersecting the region G (Fig. 4) is monotonic. The radiation heat flux across unit area in unit time is given by the following expression [1]:

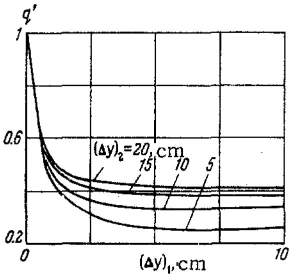

Fig. 7

$$q_w = \int_{(\omega)} d\omega \cos\theta \int_{(\Delta\lambda)} d\lambda \int_{r_0(\omega)}^{r_1(\omega)} B_\lambda(r,\omega) \exp[-t_\lambda(r,\omega)]\, dt_\lambda(r,\omega)$$

Here ω is the ray direction, θ is the angle between the ray direction and the normal to the area at C, r is the distance from C to the current point on the ray, and $r_0(\omega)$ and $r_1(\omega)$ are the distances from C along the ray with direction ω to the nearest and furthest boundaries, respectively.

We divide the region G by isothermals into $\varkappa$ elementary layers as shown in Fig. 4, and in accordance with the first transformation of § 1 we replace the continuous distribution of $T(r, \omega)$ by a stepped one so that in each elementary layer the temperature can be considered to be constant. Then

$$q_w = \int_{(\omega)} d\omega \cos\theta \sum_{j=1}^{\varkappa} z_j(\omega) B(T_j),$$

$$z_j(\omega) = \int_{(\Delta\lambda)} d\lambda B^\circ_{\lambda j} \int_{(\Delta r)_j} \exp(-t_\lambda)\, dt_\lambda$$

According to the second transformation of § 1 we can put $I_0(t)$ in the form (1.3). The coefficients a_s of the polynomial, which we denote by a_s^n, are given in § 1. Figure 2 shows $I_0(t)$ as a continuous line and approximations to it as dotted lines. Then we obtain equations for the $z_j(\omega)$ which coincide with (1.9) if in them we replace the constant coefficients a_s by the a_s^n and consider the number n_i of particles in the elementary layers to be a function of the angle ω. The integrals $M^{(jsk)}$ are computed from (1.10) and depend on the values of $n_1(\omega), \ldots, n_j(\omega)$ for the chosen direction ω and the value of $t_{(\nu)}$.

4. Examples of Numerical Calculations. Since the number n of molecules in actual problems is very large, while the cross sections σ_λ are small, to use equations of the form (1.9) for computations it is convenient to normalize the n_i and the integrals M by introducing the variables

$$n_i^\circ = n_i / K, \qquad M^{\circ(jsk)} = K^s M^{(jsk)} \tag{4.1}$$

In the numerical calculations we used the following dimensions: cm^2 for σ, cm^{-2} for n, $K = 10^{18}$.

Example 1. We compute the radiation flux incident on a wall from two-dimensional layers of a gas with thickness lying between 0.1 and 10 cm at the single temperature 13,000°K and pressure 1 atm. The relation between the cross section σ_λ of the gas and the wavelength is shown in Fig. 5 (σ, cm^2; λ, μ). In solving the problem $I_2(t)$ was approximated by a second-degree polynomial, which ensured an accuracy of 22%.

When $n = 0.057 \cdot 10^{18}$ cm^{-2}, which corresponds to $\Delta = 0.1$ cm, under the given conditions we obtained

$$M^{\circ(110)} = 0.684, \qquad M^{\circ(120)} = 1.48, \qquad M^{\circ(100)} = 0$$

Since $M^{\circ(100)}$ vanishes, we conclude that the range of wavelengths ($\Delta\lambda_m$) in which the optical thickness of the layer exceeds $t_{(\nu)} = 1.3$ is very small and does not make a significant contribution to the radiation flux. Hence for this and any smaller thickness of the radiating layer with $n \le 0.057 \cdot 10^{18}$ (cm^{-2}) $\Delta \le 0.1$ (cm) the radiation flux can be computed from (1.5) and (1.9), which in this case yield

$$q_w = \pi Bz = 4.14 \cdot 10^{-13}\, n - 3.49 \cdot 10^{-31} n^2 \ (\text{kcal} \cdot \text{m}^{-2} \cdot \text{sec}^{-1})$$

Here n is expressed in cm^{-2}.

As n and Δ increase by a factor of 100, the integrals $M^{\circ(100)} M^{\circ(120)}$ decrease monotonically while $M^{\circ(100)}$ increase monotonically. They are shown in Fig. 6a.

The radiation flux is computed from (1.5) and (1.9), while the functions $M^{(110)}$, $M^{(120)}$, $M^{(100)}$ are determined using Fig. 6a and (4.1).

For various values of the thickness Δ, Fig. 6a gives the ratio of the flux from the layer under consideration to that of an absolutely black body, $z = q_\omega/(\pi B)$. For small $\Delta \sim 0.5$ cm, z increases in proportion to Δ and for larger values of Δ the rate of increase of z diminishes; when 4 cm $\leq \Delta \leq 10$ cm z tends asymptotically to 0.62.

Example 2. We compute the radiation flux at a wall from two two-dimensional layers of a gas: one at $T_1 = 3000$°K with cross section σ_λ, as shown in Fig. 5, and the other a layer of air at $T_2 = 14{,}000$°K.

The pressures are $p_1 = p_2 = 1$ atm. When $\nu = 2$ we obtain from the equation of § 1:

$$q_w = \pi B(T_1)\left[a_1 n_1^\circ M^{\circ(110)} + a_2 (n_1^\circ)^2 M^{\circ(120)} + M^{\circ(100)}\right] + \pi B(T_2)\left[a_1 n_2^\circ M^{\circ(210)} + a_2 (n_2^\circ)^2 M^{\circ(220)} + 2a_2 n_1^\circ n_2'' M^{\bullet(221)} + M^{\bullet(200)} - a_1 n_1^\circ M^{\circ(211)} - a_2 (n_1^\circ)^2 M^{\circ(222)}\right]$$

The computed values of the integrals M° are shown in Fig. 6b, c as functions of n_1, for n_2 constant, corresponding to $(\Delta y)_2 = 20$ cm. Figure 7 shows the attenuation of the radiation flux from layer 2 due to absorption by layer 1:

$$q^\varepsilon = q_w / q_w\left[(\Delta y)_1 = 0\right]$$

as a function of the thickness $(\Delta y)_1$.

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